the mixture in the relaxation zone. The gas temperature increases more intensely due to braking. The particles heat due to heat exchange with the continuous phase, remaining colder than the gas. Increase in the relaxation time τ_1 naturally leads to freezing of the heat exchange process. At $\tau_1 \sim 0$, the change in T, T_2 to their final value occurs in a boundary layer the length of which changes little after $\tau_1 < 0.01$.

LITERATURE CITED

1. A. V. Fedorov, V. M. Fomin, and M. Kh. Okhunov, "Mathematical description of flow of a mixture of gas and liquid (solid) particles with consideration of crystallization (fusion)," in: Reports to the MSS Conference [in Russian], Tashkent (1979).
2. A. V. Fedorov, V. M. Fomin, and M. Kh. Okhunov, "Mathematical description of a mixture of gas and particles with consideration of crystallization and fusion," Preprint 8-83, ITPM Sib. Otd. Akad. Nauk SSSR (1983).
3. A. V. Fedorov, V. M. Fomin, and E. P. Chirkashenko, "Qualitative study of equations describing quasi-one-dimensional nonequilibrium flow in channels," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1983).
4. P. P. Wegener (ed.), Nonequilibrium Flows, Part 1, Marcel Dekker (1979).
5. B. L. Rozhdestvenskii and N. N. Yanenko, Systems of Quasilinear Equations and Their Application to Gasdynamics [in Russian], Nauka, Moscow (1968).
6. C. W. Gear, "The automatic integration of ordinary differential equations," Commun. ACM, 14, No. 13 (1971).
7. J. F. Clarke and J. B. Rodgers, "Shock waves in a gas with several relaxing internal energy modes," J. Fluid Mech., 21, No. 4.

TIME—FREQUENCY CHARACTERISTICS OF AN ELASTIC WAVE RADIATED BY A CAMOUFLET EXPLOSION

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UDC 534.222

One of the features of explosive action upon a medium is the radiation of elastic waves by the explosive source. This process is of interest, since the range of action of elastic waves on the medium significantly exceeds the dimensions of the destruction zone created by the explosion. Data concerning the explosion transferred by elastic waves can be received at large distances from the center. The carrier of these data is the elastic wave, the spectrum of which is usually characterized by some fundamental frequency. The characteristic frequency is determined by the dimensions of the elastic wave source R_e: $\omega_0 = c_\ell/R_e$, where c_ℓ is the speed of sound in the perturbed medium. The spectral characteristics of the elastic wave contain information on the properties of the medium surrounding the change [1]. It is thus of interest to study the effect of medium parameters in the vicinity of the explosion on the frequency—time characteristics of the radiated wave, as well as upon the seismic efficiency of an underground explosion.

We will consider the explosive process from the moment of shock wave formation. We assume that on the shock wave front the medium is compressed due to collapse of pores. The medium then breaks into particles and behind the front the medium expands due to the dilatance effect [2]. In this stage the velocity of the shock wave front or destruction wave exceeds the speed of propagation of longitudinal compression waves in the given medium. After the velocity of the front becomes equal to the velocity of longitudinal waves elastic waves begin to radiate from the destruction wave front, continuing after the latter halts.

At the initial moment a shock wave breaks away from a spherical cavity of radius a_0, filled by gas at a pressure of p_0. The increase in density of the medium at the front is defined by the quantity [3]

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$$\varepsilon(R) = 1 - \rho_0/\rho(R) = \varepsilon_0(a_0/R)^\lambda, \tag{1}$$

where R is the radius of the shock wave front; ρ_0, initial density of the medium; $\rho(R)$, density on the front. Specifying such a dependence of compaction on front radius simulates attenuation with distance from the center of the explosion. As will be shown below, the parameter λ affects the mechanical action of the explosion and the amplitude of the radiated elastic wave significantly.

The motion of the medium behind the front is described by the equations of motion, continuity, and dilation:

$$\rho \frac{du}{dt} = \frac{\partial \sigma_r}{\partial r} + 2\frac{\sigma_r - \sigma_\varphi}{r}; \tag{2}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) = 0; \tag{3}$$

$$\frac{\partial u}{\partial r} + 2\frac{u}{r} = \Lambda(\rho, \sigma_r)\left|\frac{\partial u}{\partial r} - \frac{u}{r}\right|. \tag{4}$$

Here u is the mass velocity of the medium; ρ, density; r, coordinate; t, time; Λ, dilation rate, which in subsequent calculations will be assumed constant. The relationship between the components of the stress tensor σ_r and σ_φ is given by the Prandtl expression

$$\sigma_r - \sigma_\varphi = k + s(\sigma_r + 2\sigma_\varphi) \tag{5}$$

where k and s are parameters.

We write the conditions of conservation of mass and momentum on the shock wave front in the form

$$u(R) = \varepsilon(R)\dot{R}, \quad \sigma_r(R) = -\sigma^* - \rho_0\varepsilon(R)\dot{R}^2, \tag{6}$$

where σ^* is the strength of the medium under decompression; the dot indicates differentiation with respect to time.

In Lagrangian variables system (2)-(4) takes on the form

$$\rho_0 r_0^2 r^{\alpha-2}\frac{\partial u}{\partial t} = \frac{\partial}{\partial r_0}\left[r^\alpha\left(\sigma_r(r) + \frac{k}{3s}\right)\right]; \tag{7}$$

$$\frac{\partial r}{\partial r_0} = \frac{r_0^2}{r^2}\frac{\rho_0}{\rho}; \tag{8}$$

$$\Lambda\frac{\partial}{\partial t}\ln(\rho r^3) + \frac{\partial}{\partial t}\ln\rho = 0, \tag{9}$$

where $\alpha = 6s/(2s + 1)$, $p(r_0, t) = -\sigma_r(r_0, t)$. Integrating Eq. (7), we obtain the camouflet equation describing propagation of the destruction wave and expansion of the explosion cavity.

Such a solution is valid while the front velocity $\dot{R}$ exceeds the propagation rate of longitudinal perturbations in the given medium c_ℓ. When $\dot{R} < c_\ell$, a spherically symmetric region of elastic deformations is formed ahead of the destruction zone, the front of which moves at a velocity c_ℓ. The physical quantities in this region can be expressed in terms of the reduced perturbation potential [4] $\varphi(\tau - r)\left(\tau = \frac{t}{t_0} = \frac{t}{a_0}\sqrt{\frac{p_0}{\rho_0}},\ r = \frac{r}{a_0}\right)$:

$$\sigma_r^e = -\rho_0 c_l^2\left[\frac{\varphi''}{r} + 2\frac{1-2\nu}{1-\nu}\left(\frac{\varphi'}{r^2} + \frac{\varphi}{r^3}\right)\right] - p_h, \tag{10}$$

$$\sigma_\varphi^e = -\rho_0 c_l^2\left[\frac{\nu}{1-\nu}\frac{\varphi''}{r} - \frac{1-2\nu}{1-\nu}\left(\frac{\varphi'}{r^2} + \frac{\varphi}{r^3}\right)\right] - p_h,$$

$$\rho^e = \rho_0(1 + \varphi''/r), \quad v^y = c_l\left(\frac{\varphi''}{r} + \frac{\varphi'}{r^2}\right), \quad w^y = a_0\left(\frac{\varphi'}{r} + \frac{\varphi}{r^2}\right).$$

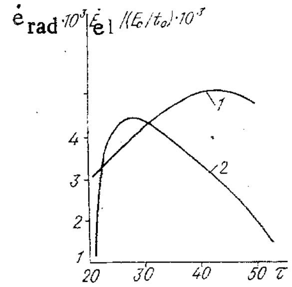

Fig. 1

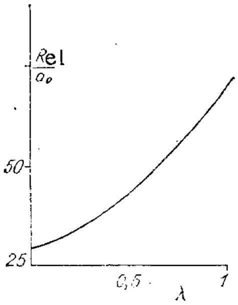

Fig. 2

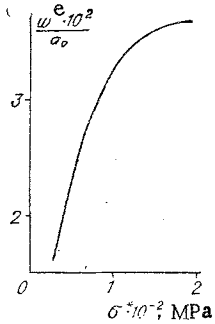

Fig. 3

The superscript e here indicates that the given quantity refers to the elastic zone; p_h is the lithostatic pressure; v^e is the mass velocity; w^e are elastic perturbations. On the boundary between the destruction zone and elastic region the medium is acted on by a decompression $\sigma_r^e(\tau - R(\tau)) = -\sigma^*$. This gives an equation for the potential

$$\frac{\varphi''}{R} = \frac{\sigma^* - p_h}{\rho_0 c_l^2} - 2\frac{1-2\nu}{1-\nu}\left(\frac{\varphi'}{r^2} + \frac{\varphi}{r^3}\right). \tag{11}$$

In the future all quantities having the dimensions of length will be dedimensionalized using the value a_0 as a reference, stresses will be normalized to p_0, velocities to $\sqrt{p_0/\rho_0}$, and densities to ρ_0. When the destruction wave is preceded by an elastic precursor, the conditions on the front, Eq. (6), have the form

$$u(R) = \dot{R}\varepsilon(R) + (1-\varepsilon)v^e(R),\ \sigma_r(R) = \sigma_r^e(R) - \rho_0\varepsilon\left[\dot{R} - v^e\right]^2. \tag{12}$$

In this case the camouflet equation obtained by integration of Eq. (7) with boundary conditions (12) can be written as

$$A\ddot{R} + B\dot{R}^2 + C\dot{R} = D,\quad A = \varepsilon(R)Y, \tag{13}$$

$$B = \left(\frac{n}{R}\varepsilon(R) + \frac{\partial\varepsilon}{\partial R}\right)Y - nR^n\varepsilon^2 X + \varepsilon\rho^e(R)R^{\alpha-n},$$

$$C = v(1-\varepsilon)\left[\frac{n}{R} - \frac{\dot{v}}{v} - \frac{1}{1-\varepsilon}\frac{\partial\varepsilon}{\partial R}\right]Y - 2nR^n v\varepsilon(1-\varepsilon)X - 2v\varepsilon R^{\alpha-n},$$

$$D = \frac{k}{3s}(R^\alpha - x^\alpha)/R^n - x^\alpha R^{-n}\sigma_r(x) - \sigma^* R^{\alpha-n},$$

$$X = \int_1^R r_0^2 r^{\alpha-3-2n}(r_0)\,dr_0,\ Y = \int_1^R r_0^2 r^{\alpha-2-n}(r_0)\,dr_0,\ x = \frac{a}{a_0},\ \sigma_r(x) = -p(x).$$

The stress on the cavity wall $\sigma_r(x)$ can be found from the condition of adiabatic expansion of the explosion gases

$$pV_c^\gamma = \text{const}, \tag{14}$$

where p is the gas pressure, V_c is the cavity volume, and γ is the adiabatic index. Simultaneous numerical solution of Eqs. (11) and (13) provides a description of the motion of the elastic and destruction waves.

We will now turn to the results of the calculations. In the case of practical application of explosions the amount of explosive energy E_{el} transferred into the elastic region is of interest.

In Fig. 1, curve 1 shows the time dependence of the quantity $\dot{E}_{el}/(E_0/t_0)$ (E_0 is the explosion energy), curve 2 is the time dependence of the elastic energy radiation rate at infinity $\dot{e}_{rad}(\tau)$. In the notation being used we have

$$\dot{E}_{el} = 4\pi R^2\sigma^* v^e(R). \tag{15}$$

The radiant energy radiation rate [5]

$$\dot{e}_{\text{rad}} = \frac{d}{d\tau}\left\{\frac{4\pi\rho_0}{c_l}\int_0^{\tau}[\varphi''(z)]^2 dz\right\}. \tag{16}$$

It is evident from Fig. 1 that each curve has a maximum, although the maximum occurs at different times. Since the elastic wave is radiated from the front of the destruction zone, it is logical to identify the dimensions of the elastic wave source with the dimensions of the destruction zone when the rate of energy radiation into the elastic region $\dot{E}_{el}$ or the rate of elastic radiation energy at infinity $\dot{e}_{rad}$ reaches its maximum. By the first criterion the radius of the elastic wave radiator R_{el} is equal to 42.40; by the second, $45.7a_0$. Since the maximum radius of the destruction zone for the given calculation is $46.2a_0$, it is evident that in the majority of cases one may take the radius of the elastic radiator equal to the final dimensions of the destruction zone.

We will now determine the dependence of the elastic radius of the seismic radiator upon parameters of the medium. To separate the effect of one or the other parameter on R_{el} all other parameters are held constant in the corresponding calculations. The majority of the calculations employed the following physical quantity values: $p_0 = 0.7\cdot10^8$ kPa, $a_0 = 3$ m, $\gamma = 1.4$, $s = 0.1$, $k = -10^4$ kPa, $\rho_S = 2.63$ g/cm³ (ρ_S is the density of the solid phase of the medium).

A study was made of the dependence of elastic wave radiator radius on porosity of the medium. Porosity is one of the factors which has the greatest effect on both source dimensions and the amount of elastic energy radiated. With increase in porosity of the medium from 1 to 25%, the radiator radius decreases from $75a_0$ to $15a_0$; i.e., by a factor of five times. Such a result is understandable if we consider that with increase in porosity the energy dissipated in the shock front increases. This leads to a decrease in dimensions of the destruction zone and the elastic radius. The effect is most significant at low porosity values (from 1 to 5%).

In the calculations it was assumed that compaction in the destruction wave front $\varepsilon(R)$ is constant, i.e., the parameter λ in (1) is equal to zero. If $\lambda > 0$, then compaction on the front decreases with distance from the center of the explosion by a power law, Eq. (1). The character of the function $R_{el}(\lambda)$ (Fig. 2) can be explained analogously to that of the function $R_{el}(m_0)$. When compaction on the destruction wave front decreases with radius, the total volume of collapsed pores decreases significantly. The energy dissipated in collapsing these pores then decreases greatly, which leads to an increase in the dimensions of the elastic source and a decrease in the characteristic frequency of the seismic signal. In this sense an explosion in a porous medium with varying compaction is equivalent to an explosion in a low porosity medium with constant compaction on the shock wave front.

It is evident from Eqs. (10) and (11) that the reduced perturbation potential $\varphi(\xi)$ and its derivatives are proportional to the strength of the medium under decompression σ^*. It follows from this that the values of stress, velocity, displacement, and density in the elastic wave are also proportional to σ^*. Figure 3 shows a graph of elastic displacement at the boundary of the destruction zone as a function of strength of the medium, calculated for the moment at which motion of the cavity and destruction wave front end. With increase in strength of the medium the elastic perturbations increase intensely.

Strength of the medium affects the dimensions of the elastic wave radiator differently. With increase in strength the resistance of the medium to shock destruction increases. As a result the dimensions of the destruction zone decrease and, consequently, the radius of the elastic source also decreases. The function $R_{el}(\sigma^*)$ is shown in Fig. 4. With increase in σ^* from $0.3\cdot10^2$ to $1.5\cdot10^2$ MPa the value of the elastic radius decreases 1.5 times. The calculations were performed for $m_0 = 0.05$, $\lambda = 1$, $\Lambda = 0.07$, $\rho_0 = 2.5$ g/cm³. This feature of the dependence of elastic radiator radius on strength of the medium also manifests itself in a change in the spectral characteristics of the source. We will use the Fourier transform of the reduced velocity potential $\varphi'(\omega)$ to characterize the spectrum. The function $\varphi'(\omega)$ can be used to define the spectral composition of any physical quantity in the elastic wave. To find $\varphi'(\omega)$ we solve the problem of explosion with radiation of an elastic wave. After the destruction wave halts, the solution of the Sharpe problem [6] is constructed for radiation of waves by a loaded sphere, upon which a constant pressure $p = -\sigma^*$:

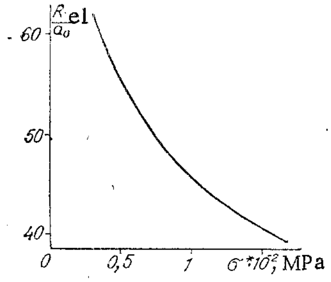

Fig. 4

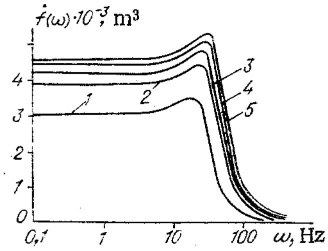

Fig. 5

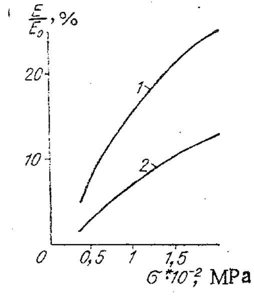

Fig. 6

$$\varphi(\xi) = R_m^3 \frac{\sigma^*}{\rho_0 c_l^2}\left[1-\left(C_1 \sin\tilde{\omega}\xi + C_2\cos\tilde{\omega}\xi\right) e^{-\frac{c_l}{2R_m}\xi}\right].$$

Here R_m is the maximum radius of the destruction zone, $\tilde{\omega} = \frac{\sqrt{1-2\nu}}{1-\nu}\frac{c_l}{R_m}$; C_1 and C_2 are constants. After taking the Fourier transforms of the potentials thus determined, curves 1-5 of Fig. 5 were obtained, corresponding to strengths $\sigma^* = 30$, 70, 110, 150, and 190 MPa. It is evident that with increase in strength the maximum of the spectrum shifts toward higher frequencies and corresponds approximately to a frequency $\tilde{\omega} = \frac{\sqrt{1-2\nu}}{1-\nu}\frac{c_l}{R_m}$, close to the characteristic frequency of the problem $\omega_0 = c_l/R_m$. With increase in strength R_m decreases (see Fig. 4), which leads to an increase in the characteristic frequency of the elastic signal. The amplitude $\varphi'(\omega)$ is proportional to $R_m^3\frac{\sigma^*}{\rho_0 c_l^2}$. Despite the fact that, in accordance with Fig. 4, the quantity R_m decreases with strength, growth in σ^* dominates over the decrease in $R_m^3(\sigma^*)$. As a result, the spectral amplitude of the reduced velocity potential $R_m^3\frac{\sigma^*}{\rho_0 c_l^2}$ increases with increase in strength of the medium for all values of σ^*. The amplitude of the elastic displacement potential decreases with increase in the strength of the medium, since the latter leads to an abrupt decrease in shock wave amplitude and, consequently, decrease in amplitude of the elastic wave generated by the latter.

As is known from experiment [7, 8], at a certain stage in the explosion breakthrough of gases from the explosion cavity into the pore space of the medium destroyed by the shock wave is possible. It is obvious that this phenomenon should lead to a decrease in the mechanical effect of the explosion. In analogy to [3], in the present study an approximate estimate was made of the effect of gas escape from the explosion cavity upon the explosion process. In other words, it was assumed that during the cavity expansion stage the gases instantaneously fill the entire volume of pores and cracks in the destruction zone. Change in strength of the medium upon filling of the pores by gas was neglected. The condition describing expansion of the explosion gases, Eq. (14), now takes on the form

$$p(V_c + V')^\gamma = \text{const}, \tag{17}$$

where V' is the total pore volume in the destruction zone, composed of pores compressed in the shock wave front and cavities created by dilation of the medium. Calculations show that this volume, normalized to the initial volume of the explosion cavity, $V'(\tau) = m_0(R^3(\tau) - 1) - (x^3(\tau) - 1)$. It is evident from calculations performed with boundary conditions on the cavity walls in the form of Eq. (17) that consideration of gas escape from the cavity leads to a significant reduction in the dimensions of the elastic radiator. For a medium porosity of the order of 15%, the elastic radius decreases by a factor of two, while for $m_0 = 2\%$ it decreases by 2.8 times, which explains the more significant cavity expansion and decreases in gas pressure in a highly porous medium. In this case the change in pressure within the cavity has little effect on development of the explosion. Moreover, consideration of gas departure from the cavity leads to a significant reduction in elastic wave energy and elastic energy radiated at infinity. However, for real subterranean explosions, gas breakthrough can occur only in the final stage of the explosion with the gases filling the entire volume of pore space, so that these numbers are in fact somewhat lower. For low power explosions communication between the explosion cavity and the pore space is of little import.

The total elastic energy and the elastic energy radiated at infinity can be determined by integration over time of the quantities defined by Eqs. (15) and (16). Figure 6 shows $E_{el}(\sigma^*)$ (curve 1) and $e_{rad}(\sigma^*)$ (curve 2). Both quantities are expressed as percentages of the total explosion energy, i.e., curve 2 corresponds to the seismic efficiency of the elastic source. When $\sigma^* = 30$ MPa the radiated elastic energy comprises 32% of the total elastic wave energy. With increase in σ^* it increases, and at $\sigma^* = 190$ MPa, 52.5% of the elastic energy is radiated at infinity.

Thus, the dimensions of the elastic wave radiator coincide with the maximum radius of the destruction region. This value determines the characteristic frequency of the elastic wave at distances from the explosion center such that change in spectral composition due to wave attenuation still has no effect. The characteristic frequency of the elastic signal depends most significantly on the strength and initial porosity of the medium. Moreover, the character of medium compaction in the shock wave front affects the dimensions of the seismic radiator strongly. The elastic wave energy, as well as the elastic energy radiated at infinity, are determined mainly by the porosity and strength of the medium under decompression. For a more accurate evaluation of the radiated elastic energy it is necessary to consider escape of explosion gases from the cavity as it expands.

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LITERATURE CITED

1. R. A. Mueller and J. R. Murphy, "Seismic characteristics of subterranean nuclear explosions, Pt. 1, Calculation of the seismic spectrum," in: Underwater and Underground Explosions [Russian translation], Mir, Moscow (1974).
2. V. N. Nikolaevskii, "Relationship between volume and shear deformations and shock waves in weak soils," Dokl. Akad. Nauk SSSR, 177, No. 3 (1967).
3. A. A. Zverev and V. S. Fetisov, "Gas cavity expansion in a variably compacting dilating medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1982).
4. S. Z. Dunin and O. V. Nagornov, "Radiation of elastic waves in camouflet explosions," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1982).
5. S. Ya. Kogan, Seismic Energy and Methods for Its Determination [in Russian], Nauka, Moscow (1975).
6. M. A. Sadovskii (ed.), Mechanical Effect of Underground Explosion [in Russian], Nedra, Moscow (1971).
7. K. E. Gubkin, V. M. Kuznetsov, and A. F. Shatsukevich, "Heat-mass exchange in explosion in solids," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1978).
8. V. M. Kuznetsov, A. G. Smirnov, and A. F. Shatsukevich, "Mechanism of water expulsion from the near zone in an explosion in water-saturated soil," Fiz.-Tekh. Probl. Razrab. Polezn. Iskop., No. 1 (1982).